

Decimal and Binary Numbers
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As you work with the Internet and computers you will work with several number systems. Number systems give us a way to express numbers. There are an infinite number of number systems available. However we normally work with only a few, or often one or two. The three most common used with computer work is decimal, binary and hexadecimal

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Chapter 1

Numbering Systems

You already know the decimal number system. It is a base 10 number system. It is the number system that humans have standardized on. It is not the only number system that humans have used. The people in ancient Babylon used a base 60 number. As a result of them using that system we still have some measurements from it. An hour has 60 minutes and a circle has 360 degrees because of this.

The base number of a number system is how many single digit numbers are in that number system. The base 10 (decimal) number system has 10 single digit numbers. They are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The base 60 number system has 60 single digit numbers. For the purposes of this book we will not figure out what all the symbols they used were. The base 2 (binary) number system has two single digit numbers. They are 0 and 1.

We use the base 10 number system because we can relate our bodies to it. How did you originally learn to count? You probably first used your fingers. How many fingers (including thumbs) does a person normally have? Ten. When you ran out of fingers you either made a mark on the ground or on paper and restarted, with that mark representing 10. Each of your fingers represented a single digit number. The mark on the ground or paper represented the 10s column. We then developed symbols for these ten digits, which we use 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Once you use the ten single digit numbers, you then use the next column to show you have gone beyond the highest single digit number (9). So we then have the one's,

ten's, hundred's, etc. columns from right to left. The value of each of these columns is a power of the base number of the number system. So in the base ten number system the first column is the 10^0 column. Any number raised to the power of zero is always 1. The next column is the 10^1 column. Any number raised to the power of one is the value of it's self, in this case 10. The next column is the 10^2 column. This means it is 10 squared or 10 times 10. The value of this column is 100. This logic continues for all columns.

Computer Binary

Computers use the binary numbering system. Computers are electrical/electronic devices. It is easy to represent anything in an electrical/electronic environment as either ON or OFF. It is harder to do the representation as ten single digit numbers as trying to decide how much On or Off is difficult. The light switch on your wall probably has two positions, ON or OFF. Likewise in the computer if we could represent everything as ON or OFF it would be much easier. For this reason the computer uses binary to store every thing.

We can then group these binary digits, called bits, together. Normally these are grouped in collections of eight bits that are called a byte. A byte can be used to represent a single character, which you are used to, by using either the EBCDIC or ASCII coding schemes. To be able to represent newer symbols, such as the Euro, and Asian languages, the UNICODE coding scheme is often used which requires two bytes to represent a character, but logically works the same.

IP Addresses

Internet Protocol (IP) version 4 addresses will consist of numbers. They use four one byte fields to represent an address in version 4. Each of these one byte fields can range from 0 to 255 in decimal, but are each a one byte binary field that ranges from 00000000 to 11111111. Therefore you can have IP addresses such as 192.168.67.1.

Chapter 2

Binary Numbers

The binary numbers work on the same approach as decimal numbers. The binary number system is a base 2 number system. There are only two digits in binary, 0 and 1. The binary number system works logically the same as the decimal number system, but uses 2s instead of 10s.

The column values in binary are based on the powers of 2, just as we the column values in the decimal number system. Starting from the right, the first column value is 2^0 or 1. (Remember a number raised to the power of zero is 1). The next column value will be 2^1 or 2. The third column value will be 2^2 or 4. The eighth column value will be 2^7 or 128. Notice that each one is 2 times the previous value.

128 64 32 16 8 4 2 1

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Chapter 3

Converting Decimal Numbers to Binary

You will need to be able to convert binary numbers to decimal numbers and decimal numbers to binary numbers as you work with a computer from a technical level. As you look at IP addresses to determine their class in binary conversions will help. You will need to understand and be able to do decimal to binary conversions to do subnetting and supernetting.

To convert a number from decimal to binary you will set up first the column values for the new binary number. Keep setting column numbers from the right to the left until you get a column value greater than your decimal number. For example to convert the decimal number 106 to binary I will set the following binary column values:

128 64 32 16 8 4 2 1

We set the values to 128 as 128 is the first column value greater than 106. We will then start putting binary numbers in the next lower column from the largest column value. In this case we will start with the 64 column.

Look at your decimal number. How many times will your column value go into it? In the example, how many times will 64 go into 106? Once. The first column we are working with should now get a 1 in it. (The 64 column.) As we work through the rest of the columns, they should only be a 1 or a 0. Any other value means we have made an error. We will now place a 1 in the 64column.

1

128 64 32 16 8 4 2 1

Now subtract the column value from your number. We will subtract 64 from 106 giving us a remainder of 42.

Now we will see how many times the next column value goes in the remainder. In this case 32 in to 42. The answer is 1 so we put a 1 in the 32 column.

1 1

128 64 32 16 8 4 2 1

We then subtract the column value from the remainder getting a new remainder. In our example 42 minus 32 gives a remainder of 10. Now we see how many times the next column value goes in the new remainder. Does 16 go into 10? No. So we will put a 0 in the 16 column and move on to the next column

1 1 0

128 64 32 16 8 4 2 1

Since we entered a zero we do not have a subtraction to perform. Now we see how many times the next column value goes in the new remainder. Does 8 go into 10? Yes. So we will put a 1 in the 8 column.

1 1 0 1

128 64 32 16 8 4 2 1

We then subtract the column value from the remainder getting a new remainder. In our example 10 minus 8 giving a remainder of 2. Now we see how many times the next column value goes in the new remainder. Does 4 go into 2? No. So we will put a 0 in the 4 column and move on to the next column.

1 1 0 1 0

128 64 32 16 8 4 2 1

We will not subtract the column value from the remainder since we entered a zero. Now we see how many times the next column value goes in the new remainder. Does 2 go into 2? Yes. So we will put a 1 in the 2 column and move on to the next column

1 1 0 1 0 1

128 64 32 16 8 4 2 1

We now subtract and have zero left. ($2-2 = 0$) Since we have a remainder of 0 we will put zeroes in any remaining columns, in this case in the 1s column

1 1 0 1 0 1 0

128 64 32 16 8 4 2 1

So the binary number equivalent of a decimal 106 is 1101010.

Chapter 4

Converting Binary Numbers to Decimal Numbers

To convert a number from the binary number system to the decimal number, we will simply determine our binary column values, then add all the column values that have ones in them, and we will have the equivalent decimal number.

To convert the binary number 11011001 to decimal first determine the column values and you should get:

1 1 0 1 1 0 0 1

128 64 32 16 8 4 2 1

Now add all the columns that have a value of one in them. In this case we will add the following: 128, 64, 16, 8 and 1. The answer we get is 217, which is the decimal equivalent of 11011001 in binary.

Are you not sure if that is really correct? Then to check our work we can convert the 217 back to binary.

256 128 64 32 16 8 4 2 1

Put a 1 in the 128 column and the remainder is 89, put a 1 in the 64 column and the remainder is 25, now put a 0 in the 32 column (32 is bigger than the remainder of 25), then put a 1 in the 16 column which gives us a remainder of 9. Now put a 1 in the 8 column with a remainder of 1. A 0 will go in the 4 column (4 is greater than 1), a 0 will

go in the 2 column (2 is greater than 1) and a 1 in the 1 column leaving a remainder of zero with no columns left. So a decimal 217 is equivalent to a binary 11011001.

1 1 0 1 1 0 0 1

256 128 64 32 16 8 4 2 1

Chapter 5

Summary

You have learned about what different number systems are and that the base number is the number of single digits in that number system. In the binary number system, there are two single digit numbers, 0 and 1. You have also learned to convert binary numbers or decimal numbers and decimal numbers to binary numbers by knowing the column values.

Appendix

Questions

1. Hexadecimal is a base 16 number system. How many single digit numbers are in it?
2. Octal is a base 8 number system. How many single digit numbers are in it? What are they?
3. Why do computers use the binary number system?

Conversion Questions

Convert the following decimal numbers to binary:

18

100

255

127

15

109

224

10

99

254

Convert the following binary numbers to decimal numbers:

00110011

10101010

11111111

00000001

01110010

11001100

11110001

11111001

10111111

00001111

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